February 1st, 2020

Machines Division C Exam

Yale Science Olympiad

Time: 40 minutes **Resources:** one binder of any size, two stand-alone calculators of any type

- Write your team number on every page.
- You may rip up the test, so long as all pages are returned in order.
- This test is organized in the following manner.
  - Section 1 contains 20 multiple choice questions. Each question is worth two points.
  - Section 2 contains 8 short answer and open response questions, each with multiple parts.
- All answers must be to the **correct number of significant figures** and with the **correct units**.
- You may assume $g = +10.0\,\mathrm{m\,s^{-2}}$ unless otherwise specified.

Team Name: _____ **KEY** _____ **Team Number:** _____

Competitor Names: _____

Good luck!

For official use only:

Section:	1	2	Total
Points:	40	60	100
Score:			

This page was once a tree, and once it was blank. Now it is neither. You can make it less blank if you like. Turning it back into a tree is more effort, but also more worthwhile.

Section 1. Multiple Choice

All answers must be written clearly and legibly within the boxes provided. No credit will be provided for illegible responses.

1. (2 points) Which of the following is an example of a second-class lever?

   A

   A. Lifting wheelbarrow. B. Using a pair of scissors. C. Riding on a seesaw.
   D. Using a fishing pole. E. Digging with a shovel.

2. (2 points) Which of the following will increase the ideal mechanical advantage of a wedge?

   B

   A. Increase its width. B. Increase its length. C. Decrease its length.
   D. Increase its hardness. E. Blunt the tip.

3. (2 points) An inclined plane has an ideal mechanical advantage of 5.00. What is the angle of the incline with respect to the horizontal?

   B

   A. 11.3° B. 11.5° C. 39.3°
   D. 78.5° E. 78.7°

4. (2 points) A wheel has an outer circumference of 1.0 m, and its axle has a diameter of 5.0 cm. Due to frictional losses, however, the apparatus only has an efficiency of 75%. What is the *ideal* mechanical advantage?

   D

   A. 0.094 B. 0.16 C. 6.1
   D. 6.4 E. 10.6

5. (2 points) It feels easier to lift a heavy load with a third-class lever because

   E

   A. it takes less time to lift it. B. the angular displacement of the load is larger. C. the vertical displacement of the load is larger.
   D. All of the above. E. None of the above.

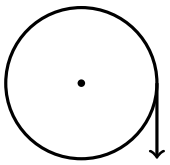
6. (2 points) Three machines are linked together to form a compound machine. The first has ideal mechanical advantage 5.0, the second 2.5, and the last 0.50. These have efficiencies 0.90, 0.80, and 0.70 respectively. What is the actual mechanical advantage of the system?

   E

   A. 1.2 B. 1.5 C. 2.0
   D. 3.0 E. 3.2

7. (2 points) Consider the following wheel being acted upon by a force.

   A

   What is the direction of the torque?

   A. Into the page. B. Out of the page. C. To the top.
   D. To the bottom. E. To the right.

8. (2 points) The ideal mechanical advantage of a block and tackle system is given by the number of

   D

   A. pulleys. B. blocks. C. total ropes.
   D. supporting ropes. E. None of the above.

9. (2 points) A car is traveling at $20\,\mathrm{m\,s^{-1}}$ when its engine shuts off. Neglecting friction, what's the maximum height it can reach?

   E

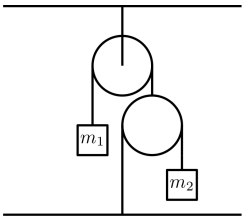
   A. 2.0 m B. 6.0 m C. 10.0 m
   D. 15 m E. 20.0 m

10. (2 points) A block of mass 10.0 kg rests on an incline of angle 20.° relative to the horizontal. If the coefficient of static friction is 0.50, what force is necessary to keep the block at rest?

    A

    A. 0 N, no direction B. 13 N, up the incline C. 13 N, down the incline
    D. 26 N, up the incline E. 26 N, down the incline

11. (2 points) Consider the following pulley system:



What mass m_2 will cause the system to be in static equilibrium, in terms of m_1?

A. $m_2 = m_1/2$ B. $m_2 = m_1$ C. $m_2 = 2m_1$
D. $m_2 = 3m_1$ E. $m_2 = 4m_1$

A

12. (2 points) Alex and Sam are playing on opposite ends of a 2 m long seesaw and weigh 60 kg and 80 kg respectively. How far away from Alex should the fulcrum be if they want to remain in equilibrium?

A. 0.43 m B. 0.86 m C. 1.0 m
D. 1.1 m E. 1.6 m

D

13. (2 points) A spur gear and a rack gear have 24 and 96 gears respectively, with the rack gear being 0.5 feet long. How many rotations of the spur gear are required to traverse the entirety of the rack gear?

A. 0.5 B. 1 C. 2
D. 4 E. 8

D

14. (2 points) A 10.0 cm long bolt has radius 0.50 cm and an ideal mechanical advantage of 50.. How many grooves does it have?

A. 100 B. 120 C. 140
D. 160 E. 180

D

15. (2 points) A certain fire hose shoots water at $25\,\mathrm{m\,s^{-1}}$. The water leaves the hose in a circular stream 3.0 cm in diameter. What is the minimum power required create such a stream? Assume that water has density $\rho = 1000\,\mathrm{kg\,m^{-3}}$.

A. 5500 W B. 6600 W C. 7700 W
D. 8800 W E. 9900 W

A

16. (2 points) The work done by friction on some object moving from point A to point B depends on what? You may assume A and B are in the $x-y$ plane.

A. The distance AB B. The horizontal component of AB C. The precise path length from A to B
D. Only the force needed to move the object E. The time needed to move the object

C

17. (2 points) Suppose machine A has twice the power of machine B. Which of the following is true?

A. A is twice as fast as B B. A and B take the same time C. A is four times as fast as B
D. B gets the job done twice as fast as A E. Same amount of time, but it's easier for A

A

18. (2 points) The Ljubljana Marshes Wheel is the oldest wooden wheel ever discovered. Where is Ljubljana?

A. Serbia B. Slovakia C. Slovenia
D. Switzerland E. Syria

C

19. (2 points) The Greek Antikythera mechanism made use of which of the following simple machines?

A. Levers B. Inclined planes C. Wedges
D. Gears E. Pulleys

D

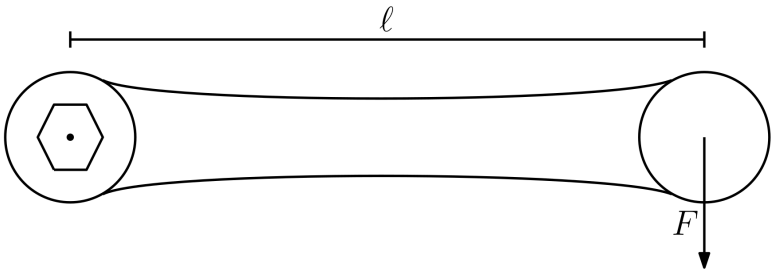
20. (2 points) Which of the following is not an advantage of a spur gear?

A. Low noise at high speed B. Simpler manufacturing C. Greater compatibility
D. Lack of axial trust E. Simplicity of design

A

Section 2. Short Answer and Related Questions

21. The bolts on the cylinder head of certain engines require tightening to a torque of $80.0\,\mathrm{N\cdot m}$. The following figures illustrates a wrench with which you want to tighten the bolts. A bolt is shown on the left side. Express your answers to two significant figures and include the appropriate units when necessary.



(a) (2 points) What two classes of levers can you find in this arrangement, assuming that F represents the effort.

Class 1 and 2

(b) (3 points) If the wrench is $\ell = 28\,\mathrm{cm}$ long, what force perpendicular to the wrench must the mechanic exert at its end?

290 N

(c) (3 points) If the six-sided bolt head is 15 mm across on average, estimate the force applied near each of the bolt's six points by a socket wrench.

1800 N

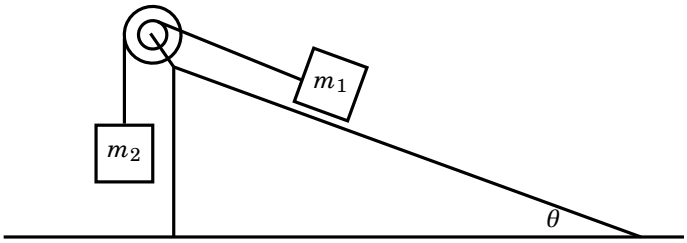
(d) (2 points) What is the ideal mechanical advantage of this wrench?

37

Rubric:

- For (a), award **(1 pt)** for each correct answer. If "Class 3" is given, max 1 point if they get at least one of the others.
- For (b), award **(2 pt)** for the a value between 280 and 300, and **(0.5 pt)** for 2 significant figures and **(0.5 pt)** for being in Newtons or equivalent.
- For (c), award **(2 pt)** for the a value between 1700 and 1900, and **(0.5 pt)** for 2 significant figures and **(0.5 pt)** for being in Newtons or equivalent.
- For (d), award **(1.5 pt)** for between 36 and 38, with **(0.5 pt)** for 2 significant figures. Some students may forget to multiply back by 6, so there might be a few 5.9-6.3. Give **(0.5 pt)** for an answer between 5.9 and 6.3 if significant figures and units are correct. Otherwise, award no points.

22. Consider the following system in which a pulley system rests on an inclined plane of angle θ.



The mass m_1 is attached to a pulley of radius R_1. This pulley shares an axis with one of radius R_2, which is subsequently attached to m_2. We have $3R_2 = 4R_1$ and $3m_2 = m_1$. The mass m_2 is considered the load.

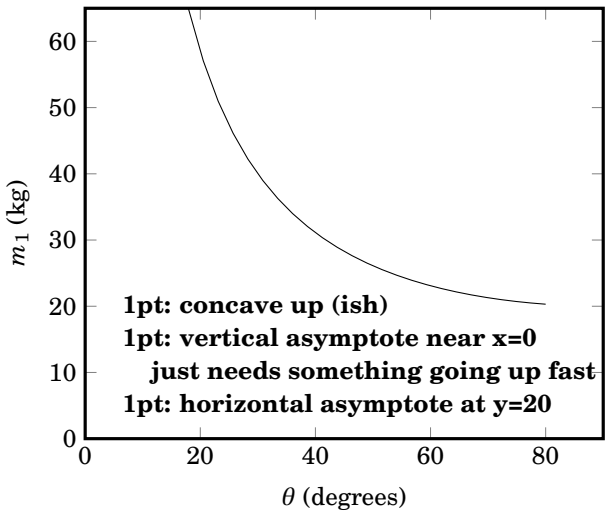
(a) (2 points) What angle θ is necessary to keep the system in static equilibrium, to three significant figures?

26.4 degrees

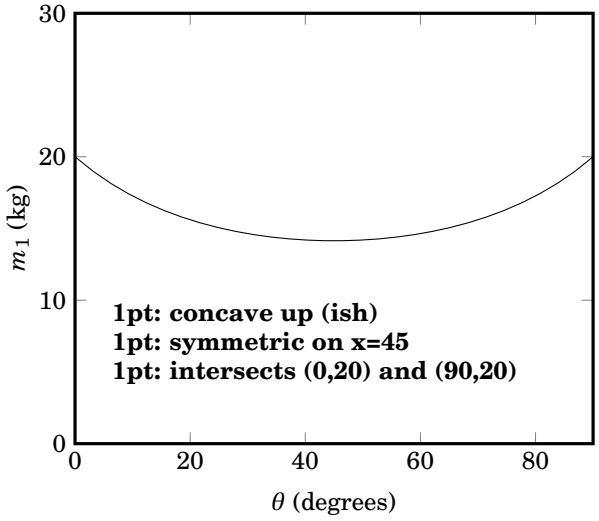
(b) (2 points) What is the ideal mechanical advantage of the system in (a)?

3

(c) (3 points) Let m_2 = 15 kg, and let m_1 be variable. Qualitatively graph θ vs. m_1 such that the system is in static equilibrium.



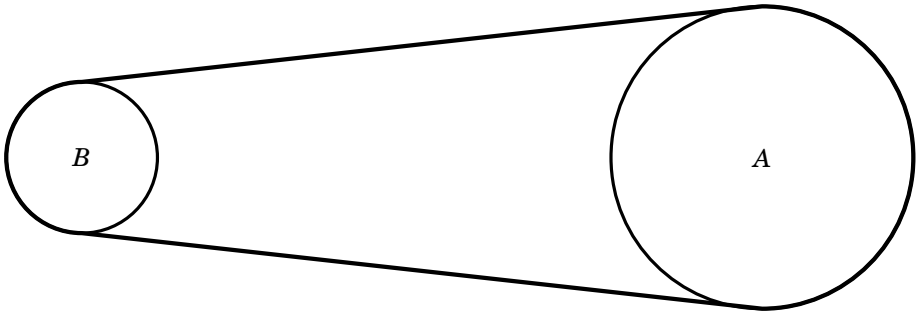
(d) (3 points) Now suppose that the inclined plane has coefficient of static friction $\mu_s = 1$. Assuming that m_2 remains 15 kg, qualitatively graph θ vs. m_1 such that the system is in static equilibrium.



Rubric:

- For (a), award **(1pt)** for the correct value, **(0.5pt)** for three significant figures, and **(0.5pt)** for degrees. Radians are acceptable.
- For (b), award **(2pt)** for the correct value. Decimals are acceptable.

23. A bicycle can be thought of as two gears connected by a chain.



The effort is being supplied via pedaling onto A.

(a) (2 points) Let N_A and N_B be the number of teeth on gears A and B respectively. Using these quantities, derive a formula for ω_A/ω_B, where ω_A and ω_R are their respective angular velocities.

$\boldsymbol{N_B/N_A}$

(b) (2 points) Suppose $N_B = 52$ teeth and $N_A = 13$ teeth. Which gear will have a greater *tangential* velocity?

Neither.

(c) (3 points) Using the values given in (b), calculate the ideal mechanical advantage. Would it be more difficult to go up a hill in this configuration than if the gears were swapped? Explain.

- **The ideal mechanical advantage would be $N_B/N_A = 4$ (1 pt).**
- **It would be more difficult if the gears were swapped (1 pt).**
- **The reverse configuration would have an IMA less than 1, so less force is extracted. (1pt)**

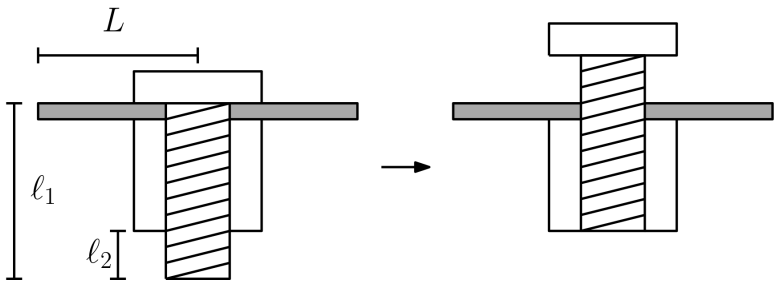
(d) (3 points) Suppose B is connected to a large wheel of radius r_C. Find the ideal mechanical advantage of the compound system in terms of r_A, r_B, and r_C.

$\boldsymbol{r_C/r_A}$

Rubric:

- For (a), award **(2 pt)** for the correct answer.
- For (b), award **(2 pt)** for the correct answer.
- For (c), award **(1 pt)** for the answer of 4. **(1 pt)** for recognizing that the current configuration is easier to go up the hill. **(1 pt)** for the reasoning.
- For (d), award **(3 pt)** for the correct answer.

24. In the movie, *Pirates of the Caribbean: Dead Man's Chest*, Captain Davy Jones of the Flying Dutchman wants to release the Kraken. To do so, he uses a screw to lift up a column, which then falls down vertically to make a loud boom. This boom calls the Kraken. The following figures are not to scale, with the first depicting the screw when it is at rest and the second when it is about to be released.



The gray bars represent the wheel spokes which rotate around the central axis and lift the screw. There are eight spokes on the wheel. Let $L = 1.5\,\text{m}$, $\ell_1 = 0.90\,\text{m}$, and $\ell_2 = 0.20\,\text{m}$. Friction is negligible.

(a) (2 points) If there are $N = 25$ threads on the screw, what is the pitch of the screw?

0.036 meter

(b) (2 points) If someone pushes on each spoke with a force of 100 N, what is mass of the heaviest screw they can lift?

21000 kg

(c) (3 points) What is the work necessary to move the screw from rest position to release position if the screw has the mass calculated in (b)?

41000 J

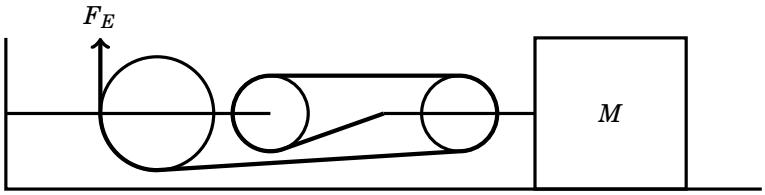
(d) (3 points) What is the end velocity of the screw in (c) after it has been released, in meters per second?

2.0 m/s

Rubric:

- For (a), award **(1 pt)** for the answer between 0.32 and 0.40 or equivalent. **0.5 pt** for 2 significant figures, and **0.5 pt** for units in meters or equivalent
- For (b), award **(1 pt)** for the answer between 20000 and 22000 or equivalent. **0.5 pt** for 2 significant figures, and **0.5 pt** for units in kilograms or equivalent
- For (c), award award **(2 pt)** for the answer between 39,500 and 42,500 or equivalent. **0.5 pt** for 2 significant figures, and **0.5 pt** for units in Joules or equivalent
- For (d), award **(2 pt)** for the answer between 1.9 and 2.1 or equivalent. **0.5 pt** for 2 significant figures, and **0.5 pt** for units in meter/second or equivalent

25. Consider the following pulley system.



(a) (1 point) What is the ideal mechanical advantage of this system?

3

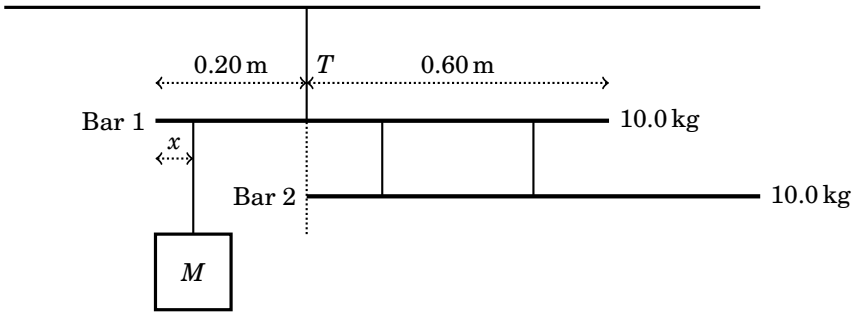
(b) (2 points) Suppose that $M = 10.0\,\text{kg}$, $\mu_s = 0.60$ and $\mu_k = 0.30$. If the mass is originally at rest, what force F_E is necessary to start moving the mass?

20 N

(c) (2 points) Assuming the same values in (b) and an overall efficiency of $\eta = 0.60$, what force F_E is necessary to start moving the mass?

33 N

26. Consider the following arrangement of uniformly dense levers. The configuration itself is rigid.



Both bars have the same length.

(a) (2 points) What is the horizontal distance between the bars' combined center of mass and the left end of Bar 1?

0.50 meters

(b) (2 points) If the system is in static equilibrium and $M = 40.0\,\text{kg}$, what is x? Give your answer in meters.

0.050 meters

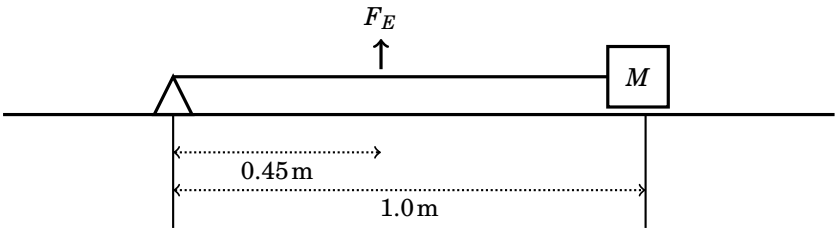
(c) (1 point) What is the tension in the topmost rope, T?

600 N

Rubric:

- For 25(a), all or nothing.
- For 25(b), **(1pt)** for the value, **(1pt)** for newtons.
- For 25(c), **(1pt)** for the value, **(0.5pt)** for newtons, **(0.5pt)** for two significant figures.
- For 26(a), **(1pt)** for the value, **(0.5pt)** for meters, **(0.5pt)** for two significant figures.
- For 26(b), **(1pt)** for the value, **(0.5pt)** for meters, **(0.5pt)** for two significant figures.
- For 26(c), all or nothing.

27. Consider the following lever system.

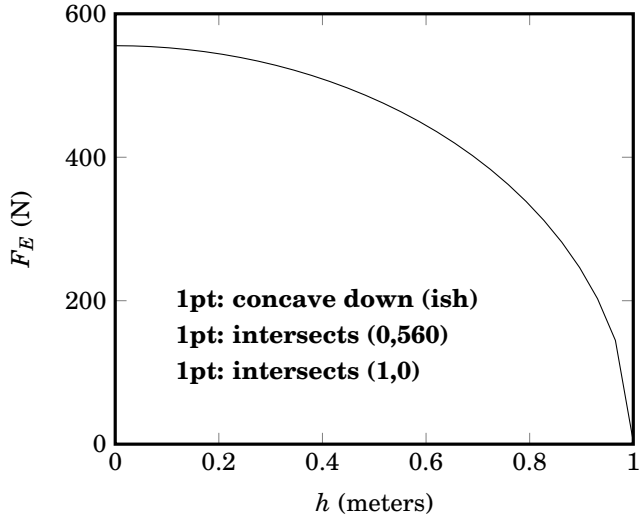


We have that $M = 25.0\,\text{kg}$, and the mass is securely attached to the bar.

(a) (2 points) What force F_E is necessary to keep the lever in static equilibrium?

560 N

(b) (3 points) The lever is rotated counter-clockwise at constant angular velocity to vertical. Qualitatively graph h vs. F_E, where h is the height above the floor of the mass.



28. Vlad is sticking a stake into the ground, which can be approximated as an isosceles triangle with apex angle $\theta = 20°$. The separation between the edged surfaces is 10.0 cm. The wedge is pushed fully into the ground and has mass 2.0 kg.

(a) (1 point) What is its ideal mechanical advantage?

2.9

(b) (2 points) When the stake is being pushed down, the ground exerts a constant force of 100 N. What is the work done by the ground on the stake?

-29 J

(c) (2 points) Suppose Vlad threw the stake into the ground. What initial velocity would be required?

5.4 m/s

Rubric:

- For 27(a), **(1pt)** for correct value (550 is also okay), **(1pt)** for units.
- For 28(a), all or nothing.
- For 28(b), **(1pt)** for the value, **(1pt)** for the sign.
- For 28(c), **(1pt)** for the value, **(0.5pt)** for significant figures, **(0.5pt)** for units.